INFLUENCE OF THE ENTROPY LAYER ON NONSTATIONARY PERTURBATION PROPAGATION IN A BOUNDARY LAYER WITH SELF-INDUCED PRESSURE

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Investigations on the stability of viscous flows were performed in [1-3] on the basis of the theory of free interaction. A theory with free interaction is used in [4] to investigate nonstationary hypersonic viscous gas flows with an entropy layer. A dispersion relation is derived, the role of the entropy layer in the nature of nonstationary perturbation propagation in the boundary layer is clarified in the case when the frequency and wavenumber take on purely real values.

The investigation formulated in [4] is continued in this paper, when the wavenumber and frequency can take on complex values also.

Let us consider the nonstationary free interaction of a boundary layer with an external hypersonic stream having an entropy layer. Following [5-8], we can write the system of asymptotic equations in the form

$$\begin{gathered}\partial u/\partial t + u\partial u/\partial x + v\partial u/\partial y = -\partial p/\partial x + \partial^2 u/\partial y^2,\\ \partial u/\partial x + \partial v/\partial y = 0,\ \partial p/\partial y = 0,\end{gathered} \tag{1}$$

if we use the similarity transformation indicated in [4], where x, y are Cartesian coordinates, u, v are the velocity-vector components, p is the pressure, and t is the time. Here both the independent variables and the desired flow parameters are taken in a dimensionless system of units.

The boundary conditions for the problem (1) are:

$$u = v = 0 \text{ for } y = 0; \tag{2}$$

$$u \to y + A(t, x) \text{ for } y \to \infty; \tag{3}$$

$$p = -\partial A/\partial x - N\partial p/\partial x, \tag{4}$$

where N is the similarity parameter characterizing the role of the entropy layer in the interaction process [8]. The boundary conditions upstream are not formulated here since the perturbed flow domain, separated from it by the characteristic x = const, can precede the gas motion under consideration.

As is customary in stability theory, the solution describing the free viscous fluid oscillations is represented in the form

$$p = \alpha e^{\omega t + kx},\ u = y - \alpha e^{\omega t + kx}\,\partial f(y)/\partial y,\ v = \alpha k e^{\omega t + kx} f(y),$$

where α is the perturbation amplitude. Linearizing with respect to the perturbation amplitude α reduces the problem (1)-(4) to the form

$$\begin{gathered}d^3 f/dy^3 - (\omega + ky)df/dy + kf + k = 0,\\ f(0) = f'(0) = 0,\ df/dy \to (1 + Nk)/k \text{ for } y \to \infty.\end{gathered} \tag{5}$$

The frequency ω and wavenumber k are related by the dispersion relationship

$$\frac{dAi\left(\frac{\omega}{k^{2/3}}\right)}{dz}\left[\int_{\omega/k^{2/3}}^{\infty} Ai(z)\,dz\right]^{-1} = -\frac{k^{4/3}}{1 + kN^2} \tag{6}$$

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Ai(z) is the Airy function of the complex variable $z = \omega/k^{2/3} + k^{1/3}y$.

The problem (5), (6) differs from the problem solved in [7] by the factor $1/(1 + kN)$ in the dispersion relation (6).

As $N|k| \to 0$ the problem (5), (6) goes over into the problem of a nonstationary boundary layer interacting freely with an external supersonic flow. For the latter it is shown in [1] that the free interaction of internal waves being propagated in a boundary layer is stable.

As $N|k| \to \infty$ the right side of the dispersion relationship (6) has the form $-k^{1/3}/N$.

As in [2], certain properties of the solution can be indicated at once that permit a judgment about the stability of the motion under consideration in problems (5), (6) with right side $-k^{1/3}/N$ in the dispersion relationship (6).

Firstly, a denumerable set of roots located in the vicinity of the negative real semi-axis corresponds to each given k (or ω) in the complex plane $\xi = \omega/k^{2/3}$. Secondly, out of all the roots with pure imaginary values of k there are modes for which the real part of ω can take on both negative and positive values. All the roots with imaginary values of k are found by a simple conversion of the analogous solutions from the theory of free interaction of a boundary layer with an incompressible fluid flow near a plate [1], when the quantity $\mp ik^{4/3}$ is in the right side of the dispersion relationship. The conversion formulas have the form

$$|k_2| = |k_1|^4 N^3,\ \omega_2 = \omega_1(|k_1|N)^2,$$

where the values $|k_1|$, ω_1 are taken from [1].

When the real part of ω equals zero, Tollmien–Schlichting traveling waves then originate, in which neutral fluid oscillations occur with constant amplitude in the time.

If the real part of the frequency is divided by $(N|k|)^{1/2}$, while the absolute value of the wave number is divided by $(N|k|)^{3/4}$, and a dependence of the reduced frequency on the reduced wavenumber is constructed, then all the dependences of the real part of the frequency on the absolute value of the wave number shrink to one curve (Fig. 1) independently of the value of N.

According to calculations, the number corresponding to the neutral oscillations is $|k_*| = 1.005^4 N^3$ (Fig. 2, solid line). The dashed line in Fig. 2 corresponds to the value $N|k| = 1$. The theory elucidated above is invalid below this curve. The vertical dashed line separates those values of N (to the left of the dashed line) for which there are no neutral oscillators.

All the perturbations with wave numbers above the values of the dashed curve will be unstable, while those below will be stable. In this paper, the continuous passage from stable to unstable perturbations is not constructed for values of N less than N = 0.995 (the intersection of the vertical dashed line with the solid curve). But it can be said that the growth of the number N (the magnitude of the damping) first results in a loss of the stability of the

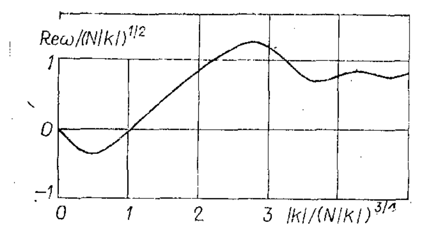

Fig. 1

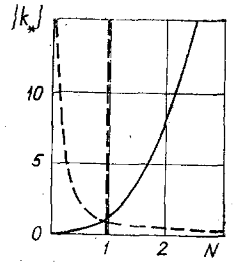

Fig. 2

longer wavelength perturbations, and then the shortwave perturbations. The latter is in qualitative agreement with the data of an experimental investigation of the influence of the bluntness of a body leading edge on the stability of boundary layer flow with an external supersonic stream. The state in investigations of perturbation development processes at supersonic velocities, and in particular, the role of body leading edge bluntness in the loss of flow stability in the boundary layer, is examined in detail in [9].

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HEAT AND MOMENTUM TRANSFER IN A TURBULENT BOUNDARY LAYER ON A CURVED SURFACE

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It is known [1-11] that the presence of relatively small streamwise curvature can have significant effect on turbulent heat and mass transfer and skin friction. Here the consideration of only the deformation of boundary layer, characterized by the ratio of boundary layer thickness to radius of curvature δ/R, leads to an appreciably lower effect of curvature on skin friction and heat transfer [2, 12] when compared to experiment. Prandtl [1] was one of the first to show the similarity between the effects of buoyant forces in stratified fluid and streamline curvature in boundary layer. He used mixing length hypothesis to suggest the following relation for turbulent skin friction: $\tau/\tau_0 = \sqrt{1 - 0.5\mathrm{Ri}}$. The Richardson number used here as the parameter differed from its usual form for stratified fluid by the replacement of acceleration of gravity by centripetal acceleration. However experimental verification of Prandtl's hypotheses showed [8] that the observed effect is an order higher than that given by theory. Empirical relations between mixing length and boundary layer parameters and streamline curvature were used to study this problem [2-7]. The basis for these methods is the analysis of Monin and Obukhov for the computation of temperature-stratified atmospheric boundary layers. Thus, it was suggested in [2] to use different relations for modified mixing length, in particular a linear relation

$$l/l_0 = 1 - \beta\mathrm{Ri}, \tag{0.1}$$

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